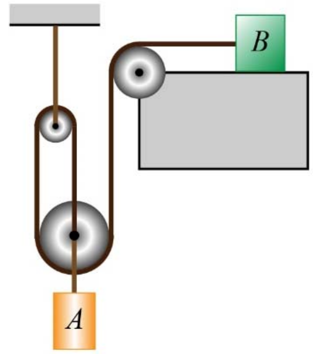
Solved Problem 5.7-4

Consider the pulley and mass system shown. If cylinder A Is released from rest determine its acceleration for the following conditions. Cylinder A and block B both have a weigh of 10 lb.

a) μ_s and μ_k between block B and the ground are 0.4 and 0.3, respectively.
b) μ_s and μ_k between block B and the ground are 0.3 and 0.2, respectively.



Given: $W_A = W_B = 10$ lb

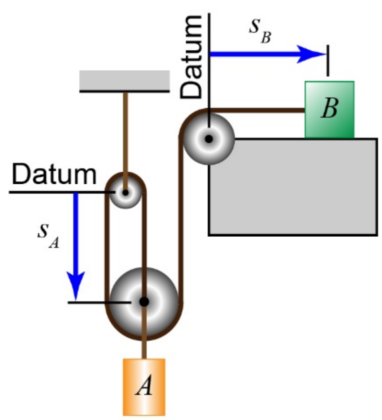
a) $\mu_s = 0.4$, $\mu_k = 0.3$
b) $\mu_s = 0.3$, $\mu_k = 0.2$

Find: a_A

Solution:

Position coordinate equation

The first step will be to determine a relationship between the acceleration of cylinder A and block B. We will do this through the position coordinate equation or by relating the length of the rope (L) to the position coordinates (s). Since there are two directions of motion we will need to use two datums, as shown in the figure. Then we will measure a distance from the datums to each moving particle. Note that we are assuming that the rope does not stretch and, therefore, $dL/dt = 0$.



$$L = 3s_A + s_B + \text{constant}$$

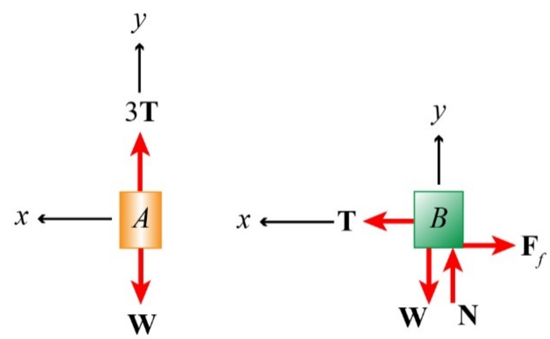
$$\frac{dL}{dt} = 0 = 3v_A + v_B$$

$$0 = 3a_A + a_B$$

$$a_B = -3a_A$$

Note that the negative relationship means that when cylinder A moves down, block B will move to the left.

Free-body diagram



The next step will be to draw a free-body diagram of cylinder A and block B. Note that I have set up the x- and y-axes to maintain the negative relationship between the accelerations of A and B found in the previous section. We will assume that the pulleys are frictionless and the tension along the rope remains constant. We don't know whether the friction exerted on block B is static or kinetic, at the moment, so we will just label it F_f. Note that I have forgone drawing a free-body diagram of the pulley connected to cylinder A. I have assumed that this pulley is massless or at least it has insignificant mass when compared to cylinder A. Therefore, the forces across the pulley balance. If the mass of the pulley was given in the problem statement, we would conclude that the mass was significant and include a FBD of the pulley.

Equations of motion

Next we will derive the equations of motion for cylinder A and block B using Newton's first and second laws. We can also avail ourselves of the relationship between a_A and a_B that we derived earlier.

Cylinder A

$$\sum F_y = m_A a_{Ay} \qquad 3T - W = \frac{W}{g}a_A = -\frac{W}{g}\frac{a_B}{3} \qquad T = \frac{W}{3}\left(1 - \frac{a_B}{3g}\right)$$

Block B

$$\sum F_y = 0 \qquad N = W \qquad F_{fs,\max} = \mu_s N = \mu_s W \qquad F_{fk} = \mu_k N = \mu_k W$$

$$\sum F_x = m_B a_B \qquad T - F_f = \frac{W}{g}a_B$$

Substituting the T solved for from the cylinder A equation we get

$$\frac{W}{3}\left(1 - \frac{a_B}{3g}\right) - F_f = \frac{W}{g}a_B \qquad F_f = \frac{W}{3}\left(1 - \frac{10a_B}{3g}\right)$$

We need to determine if block B will move when cylinder A is released. To do this we will assume that block B does not move (i.e. $a_B = 0$) and solve for the friction force (F_f) in the above equation. Then compare the friction force to the maximum static friction force ($F_{fs,max}$). If the solved friction force is greater than the maximum static friction force, then block B will move.

$F_f = \dfrac{W}{3} = 3.33$ lb when we assume that block B does not move

Part a)

For $\mu_s = 0.4$ $F_{fs,max} = 4$ lb, therefore, block B will not move.

$$\boxed{a_A = 0}$$

Part b)

For $\mu_s = 0.3$ $F_{fs,max} = 3$ lb, therefore, block B will move. We will go ahead insert the kinetic friction in the equations of motion to determine the accelerations.

$$F_{fk} = \mu_k W = 2 \text{ lb} \qquad F_{fk} = \frac{W}{3}\left(1 - \frac{10a_B}{3g}\right) \qquad a_B = 3.86 \ \frac{\text{ft}}{\text{s}^2}$$

$$\boxed{a_A = 1.29 \ \frac{\text{ft}}{\text{s}^2} \downarrow}$$