

## Solved Problem 4.3-7

The punch press shown is driven by a motor which rotates the flywheel in the counterclockwise direction at a constant 20 revolutions per minute. The flywheel is rigidly fixed to crank arm  $OA$ . Link  $AB$  is attached to the crank arm at  $A$  and a punch at  $B$ . The punch is confined to move in the vertical direction. Determine the angular velocity of link  $AB$  and the velocity of the punch when the crank is in the position shown and when the punch is at the bottom of its stroke. The radius of the crank arm from  $O$  to  $A$  is 1 foot and the length of link  $AB$  is 2 feet.

**Given:**  $l_{AB} = 2$  ft  
 $r_A = 1$  ft

**Find:**  $\omega_{AB}$ , and  $v_B$  when  $O-A$  is horizontal  
 $\omega_{AB}$ , and  $v_B$  when  $\theta = 0$

**Solution:**

*Step 1: Getting familiar with the system*

The first thing we should do is to get familiar with our system. Note that the velocity direction of  $A$  is always going to be perpendicular to the radial line drawn from  $O$  to  $A$ . This is one of the properties of velocity that we learned in the fixed-axis rotation section. Also, the problem statement says that the punch can only move in the vertical direction. Therefore,  $v_B$  is only in the  $y$ -direction. Given that  $v_A$  and  $v_B$  are both pointing downward in this position, I have assigned a clockwise direction to  $\omega_{AB}$ .

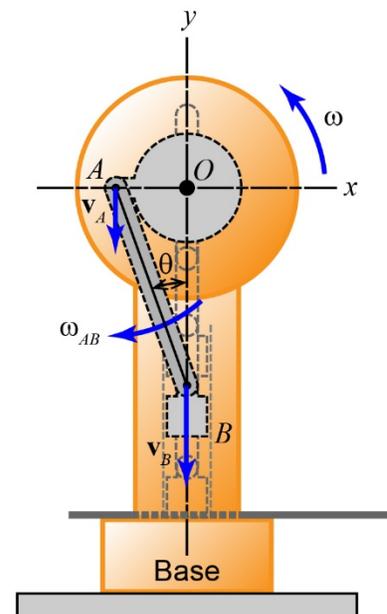
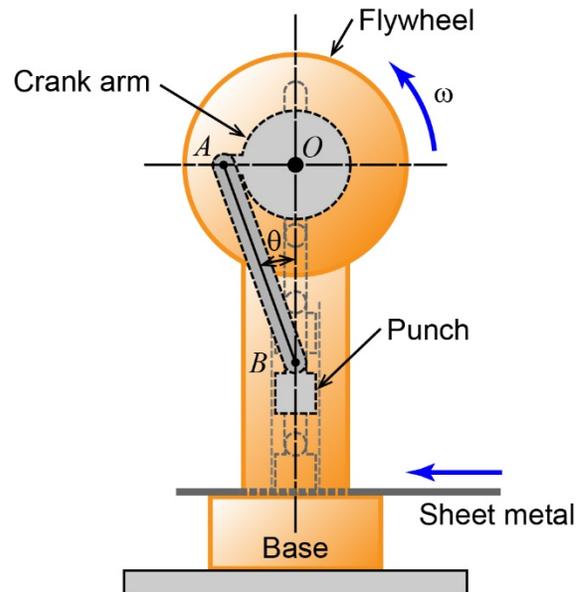
For any calculations involving angular speed or acceleration, it is always a good idea to convert their units to radians per second or radians per second squared, respectively.

$$\omega = 20 \frac{\text{rev}}{\text{min}} \frac{2\pi \text{ rad}}{\text{rev}} \frac{\text{min}}{60 \text{ s}} = \frac{2\pi}{3} \frac{\text{rad}}{\text{s}}$$

*Step 2: Analyze the velocities*

Using the fixed-axis velocity equation, the velocity of point  $A$  is

$$\mathbf{v}_A = -r_A \omega \mathbf{j} = -\frac{2\pi}{3} \mathbf{j} \frac{\text{ft}}{\text{s}}$$



Using the relative velocity equation we will analyze the velocity of the punch at  $B$ .

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -\frac{2\pi}{3} \mathbf{j} + \omega_{AB} (-\mathbf{k}) \times l_{AB} (\sin \theta \mathbf{i} - \cos \theta \mathbf{j}) = -\frac{2\pi}{3} \mathbf{j} + \omega_{AB} l_{AB} (-\sin \theta \mathbf{j} - \cos \theta \mathbf{i})$$

From the geometry we see that the angle  $\theta$  in this position is

$$\sin \theta = \frac{r_A}{l_{AB}} = 0.5 \quad \theta = 30^\circ$$

$$\mathbf{v}_B = -\frac{2\pi}{3} \mathbf{j} + \omega_{AB} 2(-\sin 30^\circ \mathbf{j} - \cos 30^\circ \mathbf{i})$$

Noting that  $\mathbf{v}_B$  has only a  $\mathbf{j}$ -component, we can extract the  $\mathbf{i}$ -component of the above equation and set them equal to zero.

$$\boxed{\omega_{AB} = 0} \quad \text{and} \quad \boxed{\mathbf{v}_B = -\frac{2\pi}{3} \mathbf{j} \frac{\text{ft}}{\text{s}}} \quad \text{when } O-A \text{ is horizontal}$$

*Step 1: Getting familiar with the system*

We will assign velocity directions with similar reasoning as we did in the first part of the problem.

*Step 2: Analyze the velocities*

Using the fixed-axis velocity equation, the velocity of point  $A$  is

$$\mathbf{v}_A = r_A \omega \mathbf{i} = \frac{2\pi}{3} \mathbf{i} \frac{\text{ft}}{\text{s}}$$

Using the relative velocity equation we will analyze the velocity of the punch at  $B$ .

$$\begin{aligned} \mathbf{v}_B &= \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \frac{2\pi}{3} \mathbf{i} + \omega_{AB} (-\mathbf{k}) \times l_{AB} (-\mathbf{j}) \\ &= \frac{2\pi}{3} \mathbf{i} - \omega_{AB} l_{AB} \mathbf{i} \end{aligned}$$

Noting that  $\mathbf{v}_B$  has only a  $\mathbf{j}$ -component, we can extract the  $\mathbf{i}$ -component of the above equation and set them equal to zero.

$$\omega_{AB} = \frac{2\pi}{3l_{AB}} \quad \boxed{\boldsymbol{\omega}_{AB} = -\frac{\pi}{3} \mathbf{k} \frac{\text{rad}}{\text{s}}} \quad \text{and} \quad \boxed{\mathbf{v}_B = 0} \quad \text{when the punch is at the bottom of its stroke}$$

