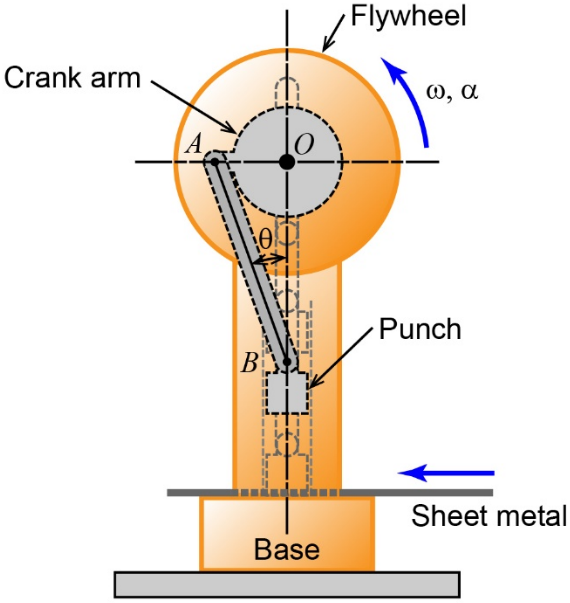
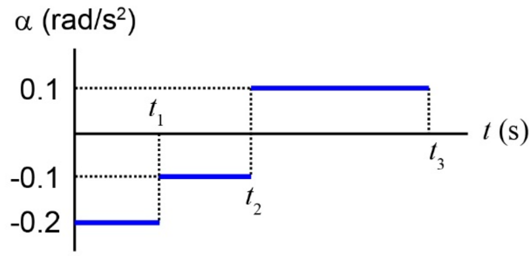
Solved Problem 4.2-7

The punch press shown is driven by a motor which rotates the flywheel in the counterclockwise direction at 20 revolutions per minute. During the punch operation, the clutch engages, disconnecting the motor, allowing the flywheel to run the press through its large inertial properties. The total punch operation takes t_1 = 1.5 seconds. After the punch operation, the flywheel rotates through a 1/2 revolution with a constant angular acceleration and then the motor engages accelerating the flywheel at 0.1 rad/s^2 until it reaches its original angular speed of 20 rpm. The angular acceleration of the punch cycle is shown in the figure provided. Find the total punch cycle time t_3.





<u>Given:</u> t_1 = 1.5 s
$\alpha_{0\text{-}1}$ = -0.2 rad/s^2, $\alpha_{1\text{-}2}$ = -0.1, $\alpha_{2\text{-}3}$ = 0.1 rad/s^2
$\omega_0 = \omega_3$ = 20 rpm
$\theta_{1\text{-}2}$ = 1/2 rev

<u>Find:</u> t_3

<u>Solution:</u>

Angular speed

For any calculations involving angular speed or acceleration, it is always a good idea to convert their units to radians per second or radians per second squared, respectively.

$$\omega_0 = 20\,\frac{\text{rev}}{\text{min}}\,\frac{2\pi\,\text{rad}}{\text{rev}}\,\frac{\text{min}}{60\,\text{s}} = \frac{2\pi}{3}\,\frac{\text{rad}}{\text{s}}$$

Using the information provided regarding the angular acceleration, we can determine the angular speed at the end of the punch operation. Note that the angular acceleration during this operation is constant allowing us to use the constant acceleration equations.

$$\omega = \alpha_o (t - t_o) + \omega_o$$

For this portion of the cycle, $\alpha_o = \alpha_{0\text{-}1}$, $t - t_o = t_1$, and $\omega_o = \omega_0$.

$$\omega_1 = -0.2(1.5) + \frac{2\pi}{3} = 1.794 \ \frac{\text{rad}}{\text{s}}$$

Angular displacement

After the punch operation the flywheel rotates through 0.5 revolutions. We will use this information plus that fact that it rotates at a constant angular acceleration to determine the time and angular speed at end of this stage. The equation we will use are

$$\theta = \frac{1}{2}\alpha_o (t - t_o)^2 + \omega_o (t - t_o) + \theta_o \qquad \omega = \alpha_o (t - t_o) + \omega_o$$

For this portion of the cycle, $\alpha_o = \alpha_{1\text{-}2}$, $t - t_o = t_2 - t_1$, and $\omega_o = \omega_1$.

$$\theta_{1-2} = \frac{1}{2}\alpha_{1-2} (t_2 - t_1)^2 + \omega_1 (t_2 - t_1) \qquad \pi = \frac{1}{2}(-0.1)(t_2 - 1.5)^2 + 1.794(t_2 - 1.5)$$

$$0 = t_2^2 - 38.88 t_2 + 118.8 \qquad t_2 = \frac{38.88 \pm \sqrt{38.88^2 - 4(118.8)}}{2} = 3.35 \text{ s}$$

$$\omega_2 = \alpha_{1-2}(t_2 - t_1) + \omega_1 = -0.1(3.35 - 1.5) + 1.794 = 1.61 \ \frac{\text{rad}}{\text{s}}$$

Punch cycle time

To determine the punch cycle time t_3, we can again apply the constant acceleration equation

$$\omega = \alpha_o (t - t_o) + \omega_o$$

For this portion of the cycle, $\alpha_o = \alpha_{2\text{-}3}$, $t - t_o = t_2 - t_3$, $\omega_o = \omega_2$, and $\omega = \omega_3$.

$$\omega_3 = \alpha_{2-3}(t_3 - t_2) + \omega_2 \qquad \frac{2\pi}{3} = 0.1(t_3 - 3.35) + 1.61$$

$$\boxed{t_3 = 8.19 \text{ s}}$$