Solved Problem 3.3-12

A car drives along a curvy mountain road. The driver uses the gas pedal to uniformly accelerate the car from 20 mph to 40 mph in 30 seconds around a constant 1000-ft radius curve. Determine the car’s acceleration as a function of time.

Given: 
\[ v_0 = 20 \text{ mph} = 29.33 \text{ ft/s}, \quad v_f = 40 \text{ mph} = 58.67 \text{ ft/s} \]
\[ \Delta t = 30 \text{ s}, \quad \rho = 1000 \text{ ft} \]

Find: \[ a \]

Solution:

Getting familiar with the problem

This problem mentions three things that should steer you in the direction of using the \( n-t \) coordinate system.

1. It mentions a curvy road. The \( x-y \) coordinate system is not ideal for use on curvy motion, especially when the \( x \)- and \( y \)-directions are coupled.
2. It mentions acceleration in the direction of motion (i.e. the driver uses the gas pedal to accelerate) or a tangential acceleration.
3. It also mentions the radius of the curve that the car is traveling on.

The first thing you should do once you have decided to use the \( n-t \) coordinate system is to write the acceleration equation down. This gives you a sense of what quantities you have, which you need to find, and most of all, it reminds you that there are two components to the acceleration.

\[ a = \dot{v}e_t + \frac{v^2}{\rho}e_n \]

Tangential acceleration

The tangential acceleration may be calculated by applying the average acceleration equation in the tangential direction.

\[ a_t = \frac{\Delta v}{\Delta t} = \frac{58.67 - 29.33}{30} = 0.98 \text{ ft/s}^2 \]

Velocity

The velocity is always in the tangential direction, therefore, we may use the constant acceleration equation (because \( a_t = \text{constant} \)) to determine the velocity as a function of time.

\[ v = a(t - t_o) + v_o \]
\[ v = a_t t + v_o = 0.98t + 29.33 \frac{\text{ft}}{\text{s}} \]
Total acceleration

Using the normal and tangential acceleration equation, we can determine the total acceleration of the car.

\[ \mathbf{a} = \dot{v} \mathbf{e}_t + \frac{v^2}{\rho} \mathbf{e}_n \]

\[ \mathbf{a} = 0.98 \mathbf{e}_t + \frac{(0.98t + 29.33)^2}{1000} \mathbf{e}_n \]

To get a sense of how much the normal acceleration increases as the velocity increases, let’s construct a plot of the tangential and normal accelerations of the car.