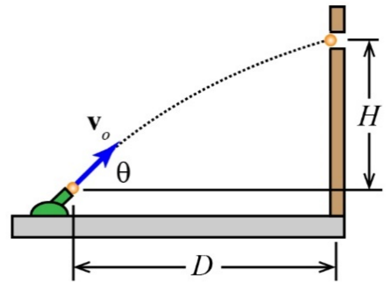
Solved Problem 3.2-10

An air gun shoots is capable of shooting a light plastic ball at various initial velocities (v_o) at various angles (θ) relative to the horizontal. It is desired to select an appropriate v_o, θ combination that would allow the ball to strike a target a horizontal distance of D and a vertical distance of H away, as shown in the figure. Derive a general equation that would allow you to calculate the appropriate v_o given θ or allow you to calculate the appropriate θ given v_o. The equation should only be a function of H, D, g, v_o, and θ. Then, calculate v_o if θ = 40º for D = 15 ft and H = 10 ft.



Given: D = 15 ft, H = 10 ft

Find: v_o if θ = 40º

Solution:

Range

Because the x- and y-directions are independent and may be analyzed as two separate rectilinear problems, the first step when analyzing a projectile problem should be to decompose the initial velocity into its x-and y-components.

$$\mathbf{v}_o = v_o(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

Neglect air resistance, the x-direction velocity is constant. We can use this fact to determine the range.

$$x = v_x t + x_o \qquad D = v_o\cos\theta\, t_B \qquad t_B = \frac{D}{v_o\cos\theta}$$

Altitude

The acceleration in the y-direction is equal to -g.

$$y = -\frac{1}{2}gt^2 + v_{yo}t + y_o \qquad H = -\frac{1}{2}gt_B^2 + v_o\sin\theta\, t_B$$

Substituting t_B obtained in the range equation we get a relationship that relates v_o and θ.

$$H = -\frac{1}{2}g\left(\frac{D}{v_o\cos\theta}\right)^2 + v_o\sin\theta\frac{D}{v_o\cos\theta} \qquad \boxed{v_o = \left[\cos\theta\sqrt{\frac{2}{g}\left(\frac{\tan\theta}{D} - \frac{H}{D^2}\right)}\right]^{-1}}$$

If θ = 40º, $\boxed{v_o = 48.8\ \frac{\text{ft}}{\text{s}}}$